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POLLUTION PERMITS AND COMPLIANCE STRATEGIES

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Pollution Permits and Compliance Strategies*

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Abstract

The paper analyzes the impact of spot and futures markets for tradeable pollution permits on the potential polluters' compliance decisions. Polluters can buy permits, invest in pollution abatement, or else stop production or source out. We show that stand-alone spot markets induce excessive investment. The introduction of a futures market reduces this incentive to invest, but is not the optimal way to control pollution. A menu of options on pollution rights, possibly coupled with intertemporally bundled sales, yields higher welfare.

Because of its focus on long-run demand elasticities and rent extraction, this paper can be applied to a variety of situations such as demand-side management, public transportation, bypass in telecommunications, or forward sales by a private monopolist.

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1 Introduction

Most developed countries have been experimenting with taxes, subsidies and markets for pollution permits to replace the old command approach to pollution control.[1] For example, title IV of the 1990 US Clean Air Act Amendments has set a cap on global emissions of sulfur dioxyde cutting pollution by more than a half from 1995 on; it defines a system of one-year spot and futures permits that are tradeable on the Chicago Board of Trade.[2] The futures price, and more generally the design of pollution control will determine the polluters' incentives to build new plants or adopt alternative, pollution-reducing technologies, namely investments in scrubbers or in boilers using low-sulfur coal or in other forms of fuel-switching (investments in gas or nuclear plants). Because such investments are in part sunk, it is important for the polluters to correctly foresee future penalties on pollution so as to plan their compliance strategy. We first study how the existing tradeable emissions permits program affects incentives to reduce pollution, and then derive the optimal mechanism.

While our primary motivation lies with the control of pollution, it must be borne in mind that the ideas developed in this section have wide applicability. They carry over to *any form of bypass that involves some long-term investment*[3] and therefore to situations in which a party with market power (the government, a private firm) must take the *long-run elasticity of demand* into account. Consider for instance the topical issue of *demand side management* in the electricity sector. Among other forms, [4] demand side management programs have an important dynamic component; for, the key decision facing rational customers is whether to invest in an alternative technology that reduces their demand for the good produced by the utility. For example, customers may switch to fuel or electricity heating or manufacturing equipment if they foresee high gas prices. The effect of expectations on bypass is also illustrated by customers purchasing automobiles when anticipating a deterioration in the quality of *public transportation*, by firms building direct access to a long-distance *telephone* company if they extrapolate high access charges to the local loop, by the possibility of banks developing *private clearing houses* at home or abroad when concerned about a high usage cost of a Central-Bank-controlled settlement

[1]See, e.g., Hahn-Noll (1983), OECD (1993), and Tietenberg (1985).

[2]For details, see, e.g. NRRI (1992), Public Utilities Report (1993), Rose-Burns (1993) and Wessler (1992). Our (1994a) paper offers a (brief) industrial organization perspective on the US reform.

[3]In a previous paper (Laffont-Tirole (1990)), we studied how bypass interferes with optimal second-degree price discrimination by a regulated firm ; because the model was static, it abstracted from the issues of commitment and intertemporal price discrimination.

[4](Pure) load management aims at providing, through, say, residential energy conservation audits, information and advice to customers on how to take advantage of peak load pricing and how to select the most efficient energy.

system, as well as by many private sector applications. Our framework also applies to the case of *forward sales by a private monopolist.*

Because of the wide applicability of our analysis, we want to abstract from the (fascinating) political economy and regulatory issues that are having a deep impact on the functioning of SO_2 pollution permits markets in the US, and instead focus on the general question of *intertemporal pricing vis-à-vis a long-term demand function.* For this reason, some of our conclusions apply more straightforwardly to the other situations described above than to the $S0_2$ pollution permits markets.

The first part of the paper (sections 2 through 4) analyzes the properties of a system of spot and futures markets for pollution permits. Stand-alone spot markets (in which the government sets at the beginning of each period the number of permits for that period) create excessive incentives for investment. The reason for this excessive bypass of the pollution permits market is that the optimal price of pollution permits is a Ramsey price, that is exceeds the marginal pollution cost in order to contribute to the reduction of the overall budget deficit. Agents do not internalize the loss of revenue they create by bypassing the market and therefore invest too much. [The other applications of our model may help clarify this point. Consumers of electricity who invest in self-generation or in energy-switching equipment do not internalize the loss of revenue they impose on the power company, which is forced to charge electricity above its marginal cost in order to balance its budget. Similarly, firms which install a direct link with long-distance telephone companies to bypass the local exchange do not internalize the local telephone company's wedge between its price and marginal cost. Last, the customers of a private monopoly do not internalize the monopoly mark-up when they invest in order to forego consumption of the monopolized good.]

This incentive to overinvest can be reduced by the introduction of a futures market. Suppose for example that the government sells in period 1 permits to pollute in period 2, at a price slightly below that that prevails under stand-alone spot markets. The second-period welfare loss is of the second-order. But the commitment to a lower second-period price for permits discourages investment, yielding a first-order gain because of the mark-up on second-period permits. We then show that the allowance program is not by itself time consistent, and discuss ways of establishing time consistency.

The second part of the paper (sections 5 and 6) derives welfare-improving reforms by considering the optimal mechanism under two alternative assumptions about regulatory instruments. Under "overall regulation", the regulator monitors not only the agents' pollution, but also their investment in bypass and their production. The optimal control then consists in offering a menu of *options* on pollution rights, the purchase price of which decreases with the striking price (these options take the form of a bilateral contract between the regulator and the agent similar to those offered on OTC markets.) Under "pure pollution regulation", the regulator (an environmental protection agency) monitors only the agents' pollution. It then may become optimal to tie the purchase of options with a lower price for current permits, giving rise to bundled sales of pollution permits. Section 7 concludes by listing some desirable extensions.

Technically, our paper builds on four literatures. From the Ramsey-boiteux analysis, we borrow the idea that pricing must participate to the coverage of the overall deficit, and therefore that the prices of pollution permits must exceed marginal cost. We then will borrow from the durable-good literature[5] when we introduce futures markets; for, the government is then a monopoly issuer of long-term pollution permits. Our analysis generalizes the standard durable-good one because the willingness to pay of the buyers of futures permits can change over time, and, mainly, because the buyers can bypass through a private investment. Accordingly, we will obtain results of independent interest such as the nonoptimality of "leasing", that is of spot markets. Section 5.1, on overall regulation, builds on Baron-Besanko (1984)'s pionneering analysis of mechanism design under commitment. It differs from their analysis in two respects. First, the investment eliminates pollution (bypass) while in their model investment increases stochastically the agents' second-period willingness to pay. Our model, besides having a different economic interpretation, is technically simpler. Second, the particular context of our model leads us to view the optimal mechanism as a simple menu of second-period options (Baron and Besanko's model is couched in a regulatory framework.) Last, we use insights from the theory of options in finance.

2 The model

• *Valuations for pollution*: The model has two periods,[6] $t = 1, 2$, and a continuum of agents/potential polluters. Each agent can consume 0 or 1 unit of pollution in each period. An agent's valuation for polluting one unit is $\theta_t \in [0,1]$. For example, θ_t is the profit obtained by producing one unit of output, thus creating one unit of pollution. Alternatively, it is the firm's opportunity cost of sourcing out for a short-run input that economizes one unit of pollution. At date 1, the agent knows her date-1 valuation θ_1, but not yet her date-2 valuation θ_2 which is learned at date 2 and is distributed according to the conditional cumulative distribution function $G_2(\theta_2|\theta_1)$, with density $g_2(\theta_2|\theta_1)$. We assume that a high first-period valuation makes a high second-period valuation more likely, in the sense of first-order stochastic dominance: $\frac{\partial G_2}{\partial \theta_1} \leq 0$. In addition to being

[5]See Fudenberg-Tirole (1991, chapter 10) for a review of this literature.

[6]The analysis can be generalized to an arbitrary number of periods.

realistic (the change in θ reflecting for example changes in demand or outage of a plant that forces greater use of dirtier plants), imperfect correlation also provides a rationale for second-period trading of pollution permits. The first-period valuation is distributed according to the cumulative distribution $F_1(\theta_1)$, with density $f_1(\theta_1)$. We make the usual assumption that $f_1/(1-F_1)$ is nondecreasing in order to ensure concavity of the social welfare function. The government does not observe the agents' individual valuation, and only observes who pollutes.

• *Investment*: At date 1, each agent chooses whether to invest in a pollution-eliminating technology. At cost i, the agent can obtain utility θ_2 without polluting.[7] In the absence of investment, the agent must (as in period 1) pollute one unit in order to obtain her valuation. Note that we assume that the investment pays off only in period 2; it is straightforward to solve the alternative case in which the investment can be used in both periods. Note last that the investment is not transferable (at least at a low cost) from one agent to another.

The private and social discount factor is denoted δ.

• *Pollution damage and welfare criterion*: Normalizing the mass of agents to be one, we let n_t denote the fraction/number of agents who pollute at date t. Let $D_t(n_t)$ denote the social damage of pollution, where $D_t(0) = 0$, $D_t' > 0$, $D_t'' > 0$. We will abstract from political economy considerations and assume that the government maximizes social welfare. The government faces a shadow cost of public funds $\lambda > 0$; that is, raising \$1 of public money costs society $\$(1+\lambda)$ because of the distortionary taxation. Alternatively, the taxes on pollution or the proceeds from sales of pollution permits could be used to finance the fixed costs of public utilities, in which case λ would stand for the shadow cost of their budget constraints.[8] [The analysis applies equally well to a situation in which the principal is a private party. What matters for our analysis is that the principal does not fully value the agents' rents.]

In the absence of investment and in a *static* context, the optimal mechanism consists,

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[7]The investment technology is modeled somewhat rigidly. There are several extensions worth considering. First, one could allow a more continuous choice in abatement (in this model, say, through a costly choice of a probability that the investment succeeds in eliminating pollution). Second, the abatement cost could vary across agents (and possibly, in the overall regulation context, be unobserved by the government); it might also depend on the agents' first-period type. Third, investments create purely private benefits; in Laffont-Tirole (1994b), we discuss investments which generate innovations in depollution technology which are public goods.

[8]This second possibility is, for example, relevant when allowances are freely allocated to utilities before being traded. [We would not expect the shadow costs of all utilities' budget constraints to be equalized, but this does not alter the nature of the argument.]

We should also point out that there is an intense debate in the US about who should benefit from the sale of emissions allowances (see, e.g., Burkhart (1993)).

as is well known, either in setting a price p or in choosing a number of emissions allowances n so as to maximize the social welfare function. (There is no distinction between quantity and price mechanisms in our model because there is no aggregate uncertainty. We will think of the mechanism as a quantity mechanism when we introduce the sale of future allowances.) For an arbitrary distribution $F(\theta)$ with density $f(\theta)$, let $n = N(p) = 1 - F(p)$ denote the static demand function and $p = P(n) = N^{-1}(n)$ the static inverse demand function. Given a pollution damage function $D(\cdot)$, the optimal number of allowances is given by:

$$\max\left\{-D(n) + (1+\lambda)np + \int_p^1 (\theta - p) f(\theta) d\theta\right\}.$$

Letting $\eta \equiv \frac{f(p)p}{1-F(p)}$ denote the elasticity of demand, the optimal price for allowances is given by the standard Ramsey formula:

$$\frac{p - \frac{D'}{1+\lambda}}{p} = \frac{\lambda}{1+\lambda}\frac{1}{\eta}. \tag{1}$$

The marginal social cost of pollution, when expressed in cost of public funds, is equal to $D'/(1+\lambda)$. Equation (1) says that the Ramsey index for the "good pollution" sold by a social welfare maximizing regulator subject to budget considerations is equal to a Ramsey fraction $(\lambda/(1+\lambda))$ divided by the elasticity of demand.

We conclude that *in a static context, the social optimum can be implemented by a market for pollution permits, with the price or the number of permits set at their Ramsey level.*

Note that we have adopted a partial equilibrium approach. We could extend (although we have not done so) our analysis to a general equilibrium situation.[9] The main concern about partial equilibrium is that it may ignore the effect of pollution taxes on the demand for inputs that may themselves be taxed. One mainly has in mind the possibility that the agents might fire some of their employees when the tax is raised. In a perfect labor market in which the employees could find a similar job elsewhere, the tax income levied on labor would not be affected. If this is not the case, the elasticities of our analysis must be replaced by (presumably higher) superelasticities,[10] which reflect a) the layoff responsiveness to a pollution tax and b) the global labor demand and supply functions. This correction is of course irrelevant in the case of a private principal.[11]

[9]See Laffont-Tirole (1993, section 3.9) for an extension of regulatory models to general equilibrium.

[10]See Laffont-Tirole (1993, chapter 3) for an exposition of the computation of such superelasticities in the context of multiple outputs.

[11]Or in case of a (nonbenevolent) regulator who does not internalize the revenue from other taxes. For example, the regulator might internalize the revenue from the pollution tax because this revenue is earmarked for specific environmental projects, but not the revenue from the other taxes which are used to reduce the budget deficit.

A related point concerns the specific application of our model to the SO_2 market. In the US, 70% of SO_2 pollution is produced by regulated utilities subject to a budget constraint. A reduction in the pollution tax relaxes the budget constraint and should therefore be valued at the shadow cost of the budget constraint. [Incidentally, let us note that the extreme subsidy implied by the US grandfathering approach amounts to a lump-sum transfer from taxpayers to regulated utilities, a policy that in principle is ruled out by regulatory statutes.]

• *Residual second-period demand curve*: For future reference, we introduce the residual second-period demand curve when the agents with valuation $\theta_1 \geq \theta$ choose to invest in period 1 and the others do not (as we will see, higher valuation agents have more incentive to invest.) The residual demand curve is then given by:

$$n_2(p_2, \theta) \equiv \int_0^{\theta} f_1(\theta_1)\,[1 - G_2(p_2|\theta_1)]\,d\theta_1. \tag{2}$$

Let $P^R(\theta)$ denote the Ramsey price given by (1) for the residual demand curve $n_2(\cdot, \theta)$. That is, $P^R(\theta)$ solves:[12]

$$\max\left\{-D_2(n_2(p_2,\theta)) + (1+\lambda)n_2(p_2,\theta)p_2 + \int_0^{\theta} f_1(\theta_1)\left[\int_{p_2}^{1}(\theta_2 - p_2)g_2(\theta_2|\theta_1)d\theta_2\right]d\theta_1\right\}. \tag{3}$$

We assume that the Ramsey price increases with θ (which is the case if the elasticity of demand is not too sensitive to θ). For future reference, we will also define the marginal cost price for the residual demand curve:

$$P^{MC}(\theta) \equiv \frac{D_2'(n_2(P^{MC}(\theta), \theta))}{1+\lambda} < P^R(\theta).$$

This marginal cost price increases with θ. In the absence of commitment and knowing that agents with $\theta_1 \geq \theta$ have invested, the government would choose price $P^R(\theta)$ in the second period.

<u>Remark</u>: Agents have unit demands for pollution in this model. Multi-unit demands give rise to second-degree price discrimination and, possibly, to marginal prices below marginal costs in the presence of bypass (Laffont-Tirole (1990)). Multi-unit demands also make allowance markets suboptimal even in a static context. In this sense our model depicts the best case for markets.

[12] We assume that the second-period welfare function is strictly concave in p_2.

3 Incentives to invest under a trading program

3.1 Stand-alone spot markets and the introduction of a futures market

The Clean Air Act Amendments of 1990 created sulfur dioxide emissions allowances, and set up a market for them at the Chicago Board of Trade. The number of allowances is already fixed.[13] To analyze the effect of a market program on investment in pollution abatement, we will first assume that the government is able to commit to the number of allowances (n_1, n_2), or equivalently (in this situation of perfect information about demand) to market prices (p_1, p_2) at date 1. We will then consider the implications of the possibility for the government of issuing new permits or of buying back existing ones at date 2.

Whether the government can commit or not to modify the number of permits in period 2, the government optimally chooses in period 1 the Ramsey price corresponding to the demand curve $n_1(p_1) = 1 - F_1(p_1)$ (see equation (1)). The focus of our analysis will naturally be the choice of the second-period price p_2.

Consider first the agents' investment decision at date 1. Given an expected second-period price p_2, an agent with valuation θ_1 invests if and only if

$$\delta E(\theta_2|\theta_1) - i \geq \delta E[\max(\theta_2 - p_2, 0)|\theta_1]. \tag{4}$$

Using first-order stochastic dominance, we see that equation (4) defines a cutoff $\theta^*(p_2)$ such that the agent invests if and only if $\theta_1 \geq \theta^*(p_2)$.[14] Furthermore, the cutoff is nonincreasing: the higher the expected price, the higher is the incentive to bypass the market. We will assume that in the relevant range $0 < \theta^* < 1$ (some bypass and others do not), and therefore the cutoff is strictly decreasing.

Including the investment costs, the second-period welfare can be written as:

$$\begin{aligned} W_2(p_2, \theta^*(p_2)) &= -D_2(n_2(p_2, \theta^*(p_2))) \\ &+ (1+\lambda) n_2(p_2, \theta^*(p_2)) p_2 \\ &+ \int_0^{\theta^*(p_2)} f_1(\theta_1) \left[\int_{p_2}^1 (\theta_2 - p_2) g_2(\theta_2|\theta_1) d\theta_2 \right] d\theta_1 \\ &+ \int_{\theta^*(p_2)}^1 f_1(\theta_1) \left[E(\theta_2|\theta_1) - \frac{i}{\delta} \right] d\theta_1. \end{aligned} \tag{5}$$

[13]In fact, the bill states that an allowance is not a property right and can be "limited, revoked and otherwise modified." The motivation for granting such ill-defined property rights seems to be the desire to leave flexibility to the government to react to macroeconomic news about demand and supply (Hausker (1992, p. 556)). There is no unanticipated macroeconomic shock in our basic model, and we will accordingly assume that allowances carry a clear and irrevocable property right. See also the discussion on aggregate shocks in this section and the next.

[14]The derivative of the difference between the left- and right-hand sides of equation (4) with respect to θ_1 is equal to $-\delta \int_0^{p_2} \frac{\partial G_2}{\partial \theta_1} d\theta_2 \geq 0$.

Let us define

$$p_2^R \equiv P^R(\theta^*(p_2^R))$$

and

$$p_2^{MC} \equiv P^{MC}(\theta^*(p_2^{MC})) < p_2^R.$$

The Ramsey price p_2^R is the rational-expectations price that would obtain if the government could not commit to a second-period price in period 1 and would lease pollution rights each period. That is, p_2^R is the price that would prevail if only spot markets were set up. Similarly, p_2^{MC} would obtain if it were known that the government (suboptimally) would use marginal cost pricing at date 2.

We now show that by committing to a lower second-period price of pollution permits the government can improve welfare. Such a commitment can be complemented in period 1 by selling pollution rights on a futures market. We denote the futures price δp_2, so that p_2 is the period-two price of the advance pollution permit. Equivalently, the government gives away to all agents options to pollute at price p_2 in period 2 [the equivalence between these two interpretations breaks down when the government's credibility is in question: see section 4].

Proposition 1 *Under commitment, the optimal emissions allowances program yields a second-period price $p_2^* \in (p_2^{MC}, p_2^R)$. In particular, the introduction of a futures market lowers the second period price from p_2^R to p_2^* and reduces bypass.*

Proof: Let us differentiate (5):

$$\frac{dW_2}{dp_2} = \frac{\partial W_2}{\partial p_2} + \frac{\partial W_2}{\partial \theta}\frac{d\theta^*}{dp_2},$$

where $\frac{\partial W_2}{\partial p_2} > 0$ if $p_2 < P^R(\theta^*(p_2))$, $\frac{\partial W_2}{\partial \theta} < 0$ if $p_2 < P^{MC}(\theta^*(p_2))$, and $\frac{d\theta^*}{dp_2} < 0$.

(a) Suppose first that $p_2 \leq p_2^{MC}$. Then $\theta^*(p_2) \geq \theta^*(p_2^{MC})$ and so $P^{MC}(\theta^*(p_2)) \geq P^{MC}(\theta^*(p_2^{MC})) = p_2^{MC}$. Hence $p_2 \leq P^{MC}(\theta^*(p_2))$ and therefore $\frac{dW_2}{dp_2} > 0$, a contradiction.

(b) Suppose next that $p_2 \geq p_2^R$. From our earlier assumption that the Ramsey price increases with θ, we have

$$p_2 \geq p_2^R = P^R(\theta^*(p_2^R)) \geq P^R(\theta^*(p_2)).$$

So p_2 also exceeds the Ramsey price for the corresponding residual demand curve. Therefore $\frac{\partial W_2}{\partial p_2} \leq 0$, and $\frac{\partial W_2}{\partial \theta}\frac{d\theta^*}{dp_2} < 0$ (since $p_2 > P^{MC}(\theta^*(p_2))$). □

Intuitively, the government commits to a lower second-period price than in the absence of commitment in order to discourage investment and limit inefficient bypass. Because pollution permits are sold at a price exceeding the marginal social cost ($p_2^R > p_2^{MC}$), a reduction in bypass increases welfare. The intuition for this can be best grasped by looking at the rational-expectations equilibrium of a *stand-alone spot market system.* The regulator then optimizes over the spot market price in period 2, and so $\partial W_2/\partial p_2 = 0$ (this is the envelope theorem). Because, furthermore, the cutoff type is indifferent between bypassing and using second-period permits, the only effect on welfare of a change in the cut off θ is the direct effect on goverment revenue and on pollution damage :

$$\frac{\partial W_2}{\partial \theta} = [(1+\lambda)p_2 - D_2'] \frac{\partial n_2}{\partial \theta}.$$

Welfare increases with the cutoff θ because $p_2^R > \frac{D_2'(n_2(p_2^R, \theta^*(p_2^R))}{1+\lambda}$.

Let us now come back to the case of a *futures* market. With a futures market, the regulator can always duplicate the stand-alone spot market solution by committing to price p_2^R. But he in general can do better by lowering the futures price p_2 below p_2^R in order to lower the cutoff $\theta^*(p_2)$. He however does not go as far as selling permits at price equal to the second-period marginal cost, because at this point there is no longer a gain to lowering the cutoff. This yields the intuition for Proposition 1.

It is sometimes suggested in the regulatory arena that prices above marginal cost create excessive bypass. For example, Costello (1992), discussing the value of demand-side management programs, argues that "the fact that electricity prices throughout many parts of the United States currently are above marginal costs by and in itself suggests that consumers are overconserving." The same point is made here in the context of pollution allowances. Our dynamic analysis further points at a way of limiting bypass in this context: The government can organize a futures market for permits and sell a large enough number of permits so as to somewhat discourage investment.

3.2 Discussion

Our treatment deserves a number of comments.

• *The spot market as a guide to investment in pollution abatement.*

Our focus is on the number of advance allowances n_2 to be put in the market. The spot number of allowances n_1 has no effect on investment and therefore is optimally set at its Ramsey level. More generally, the spot price for allowances could affect investment in the same way current electricity prices are often thought to guide investments in electricity conservation. This effect could operate through two channels in our model.

First, investments might pay off already in period 1. A low first-period price then complements a low second- period price in the fight against bypass. Second, the government might have private information about the social cost of pollution or about the aggregate demand for permits. In a situation in which the government is unable to commit to second-period prices, the government may try to signal a low second-period price through a low first-period price.

- *Time pattern of pollution and permits price.*

We can easily analyze the temporal evolution of pollution levels and permits prices in the case in which the pollution damage is time-invariant, $D_1(n) = D_2(n)$ for all n, and the agents' demand for pollution rights in the absence of investment is also constant, $F(\theta) = E_{\theta_1} G_2(\theta|\theta_1)$ for all θ. As we will discuss, these are not necessarily good assumptions. One can then show that (under commitment and for λ small) $n_1^* > n_2^*$ and $p_1^* > p_2^*$. That pollution decreases over time is a logical consequence of investments in pollution-abating technologies. That permits become cheaper over time is due to the facts that, first, agents with high demand for permits disappear from the market because they invest in pollution abatement, and, second, that a commitment to a low futures price reduces excessive bypass. While we find these two reasons compelling, their implication of a time-decreasing price may not necessarily be appealing.

The reason why we feel time-increasing prices may be desirable is that the time invariance assumptions may be violated. Consider the pollution damage. The marginal cost of pollution may increase over time for three reasons. First, growth in GNP per capita may raise the demand for pollution control. Second, scientific advances or observations may bring bad news about environmental damage. And, perhaps more cynically, an increase in environmental awareness or the organization of powerful environmental interest groups may force otherwise well-informed governments to pay more attention to the environment (in this view, the damage function $D_t(\cdot)$ is to be interpreted as the cost of pollution to politicians and not as the social cost of pollution). Third, for some pollutants the pollution damage depends on the stock rather than the flow, and the marginal costs of current pollution, namely the present discounted value of the associated future pollution increments, increases over time. Last, we note that the assumption of a time-invariant demand for pollution permits (absent investment) is also strong. The composition of GNP changes over time, and there is no reason why the environmental impact of production remains constant.

- *Aggregate uncertainty about the cost of future pollution damage.*

We have assumed that the social cost of pollution tomorrow is known, and more generally that the government can perfectly compute today the optimal level of pollution tomorrow. In practice, the government may face substantial uncertainty and may want to adjust the number of permits once the uncertainty is resolved. Scientific advances may show that the pollution damage is much greater or lower than one expected. Similarly, the price of clean technologies (such as gas) may fluctuate. This uncertainty does not invalidate the general point that the government wants to sell forward permits in order to guide investment. [It does exacerbate the time-consistency issue, as we discuss in section 4.]

Suppose that the second-period pollution damage is $D_2(n_2, \kappa)$, and that the random variable κ is publicly revealed at the beginning of period 2. The optimal contingent number of second-period permits, $n_2^*(\kappa)$, must as in the noncontingent case trade off the second-period welfare and the effect on investment behavior. Let $p_2^*(\kappa)$ denote the second-period price that will prevail (clear the spot market). Given the equilibrium cutoff θ^*, $p_2^*(\kappa)$ and $n_2^*(\kappa)$ are linked by the following relationship:

$$n_2(p_2(\kappa), \theta^*) = n_2^*(\kappa).$$

Let the government commit to intervening in the second-period spot market (by buying or selling) in order to support a total number of permits $n_2^*(\kappa)$ in state of nature κ. The cutoff is then given by

$$E(\theta_2|\theta^*) - \frac{i}{\delta} = E[\max(\theta_2 - p_2^*(\kappa), 0)|\theta^*], \tag{6}$$

where the first expectation in (6) is with respect to θ_2 and the second with respect to θ_2 and κ. Price $p_2^*(\kappa)$ contributes, for each κ, to the determination of the investment behavior as summarized by the cutoff θ^*.

Let

$$\begin{aligned} W_2(p_2, \theta^*, \kappa) \;\equiv\; & -D_2(n_2(p_2, \theta^*), \kappa) \\ & +(1+\lambda) n_2(p_2, \theta^*) p_2 \\ & + \textstyle\int_0^{\theta^*} f_1(\theta_1)[\int_{p_2}^1 (\theta_2 - p_2) g_2(\theta_2|\theta_1) d\theta_2] d\theta_1 \\ & + \textstyle\int_{\theta^*}^1 f_1(\theta_1)[E(\theta_2|\theta_1) - \frac{i}{\delta}] d\theta_1 \end{aligned}$$

denote the state-contingent second-period welfare. And let μ denote the multiplier of constraint (6). The first-order condition with respect to $p_2(\kappa)$ is:

$$\frac{\partial W_2}{\partial p_2}(p_2^*(\kappa), \theta^*, \kappa) = \mu[1 - G_2(p_2^*(\kappa)|\theta^*)] > 0.$$

As in the case of certainty, the second-period price in each state of nature is lower than the ex post optimal (Ramsey) second-period price, in order to discourage investment.

Assume further that a higher κ raises the marginal pollution damage ($\partial^2 D_2/\partial n_2 \partial\kappa > 0$). Then a standard revealed preference argument applied to the Lagrangian shows that the second-period pollution n_2^* decreases with κ.

Last, note that price reductions below the ex post optimal price are more effective, the lower the pollution damage (that is, the lower κ is.) The point is that the probability that, conditionally on not investing, the cutoff type uses a second-period permit, namely $[1 - G_2(p_2^*(\kappa)|\theta^*)]$ decreases with κ. The cutoff type is therefore more appreciative of price reductions in high-pollution states. This remark will have implications for the next section.

4 Is the allowance program time consistent?

Even if the agents' property rights are well specified, the government may in period 2 alter the number of allowances $n_2^* = n_2(p_2^*, \theta^*(p_2^*))$ by either selling new ones or buying back existing ones.

Even though the number of allowances n_2^* exceeds the Ramsey level given the second-period residual demand curve, the government has no incentive to buy back allowances. The reason is that ex post capital gains created by a reduction in the number of allowances goes to the owners of allowances, while those linked with ex ante reduction of the number raise the price received by the government. To see this, suppose that, after issuing n_2^* allowances at price p_2^*, the government buys back $n_2^* - n_2$ allowances at a price $p_2 > p_2^*$ satisfying $n_2 = n_2(p_2, \theta^*(p_2^*))$. The marginal change in welfare is equal to[15]

$$[-D_2' + (1+\lambda)p_2]\,\frac{\partial n_2}{\partial p_2} - \lambda(n_2^* - n_2).$$

The first term in this expression is the efficiency effect and, for $p_2 > p_2^*$, is strictly negative as price exceeds social marginal cost. The second term is the distributional loss associated with the transfer from the government to the owners of allowances. So, buybacks are suboptimal.

[15]Second-period welfare after a buy back of $n_2^* - n_2$ allowances at the new market price p_2 is :

$$W(p_2) = -D_2\Big(n_2(p_2, \theta^*(p_2^*))\Big) + \lambda\Big[p_2^* n_2^* + p_2[n_2(p_2, \theta^*(p_2^*)) - n_2^*]\Big]$$
$$+ \int_0^{\theta^*(p_2^*)} f_1(\theta_1) \int_{p_2}^1 \theta_2 g_2(\theta_2|\theta_1) d\theta_2 d\theta_1$$

with

$$n_2 = n_2(p_2, \theta^*(p_2^*)) = \int_0^{\theta^*(p_2^*)} f_1(\theta_1)[1 - G_2(p_2|\theta_1)]d\theta_1.$$

At date 2, the cut off type $\theta^*(p_2^*)$ is fixed and the regulator optimizes over p_2.
The marginal change in welfare $\frac{dW}{dp_2}$ is then given by the formula in the text.

In contrast, a small increase in the number of allowances at $n_2 = n_2^*$ raises second-period welfare. As in the standard durable-goods model, the issuer of permits cannot resist imposing capital losses on the owners of permits. So, the allowance program is not time consistent.

How can the government restore confidence that it will not flood the market with allowances in the future? *The standard solution to the durable-goods problem, which consists of leasing instead of selling, does not work in a context in which some consumers invest in bypass technologies.* Here, "leasing" amounts to selling allowances only on the spot market. But, as we have seen, the absence of a futures market induces excessive bypass.

In contrast, a *price support policy* (the equivalent of a "money-back guarantee" or "most-favored-nation clause" in industrial organization) allows the government to solve its time consistency problem. Suppose that the government promises to reimburse the n_2^* owners of allowances for any capital loss $(p_2^* - p_2)n_2^* \geq 0$ that they incur as the result of new issues of allowances. For $p_2 < p_2^*$, the marginal change in welfare due to a unit price decrease is

$$-[-D_2' + (1+\lambda)p_2]\frac{\partial n_2}{\partial p_2} - \lambda n_2 < 0 \qquad (7)$$

as $p_2^* < P^R(\theta^*(p_2^*))$. By insuring owners against capital losses, the government internalizes the cost of devaluing existing property rights.[16]

An alternative commitment device is to give away in period 1 free options to purchase pollution permits at price p_2^*. That is, each agent receives at price $R = 0$ an option with striking price p_2^*. Agents then do not pay for an asset that the government will later be tempted to "expropriate" by flooding the market with new copies of this asset. Ex post, the government would actually like to *raise* the permits price from p_2^* to $p^R(\theta^*(p_2^*)) > p_2^*$. However, this move is opposed by agents who were granted in period 1 the right to pollute at price p_2^*.

We summarize the analysis of this section in:

Proposition 2 *The allowance program is not time consistent in that the government would like to sell new permits in the second period. The standard method to achieve the commitment outcome, namely leasing (stand-alone spot markets) is ineffective because of the possibility of investment. The commitment outcome, though, can be achieved either through a price support policy or through a first-period giveaway of options to pollute at striking price p_2^*.*

[16]Another commitment mechanism might, as usual, be a reputation for not issuing new allowances.

- *Time consistency in the presence of aggregate uncertainty.*

Suppose now that the second-period pollution damage $D_2(n_2, \kappa)$ is state contingent and that the realization of the random variable κ occurs at the beginning of period 2 (see subsection 3.2). It is straightforward to obtain time consistency if κ is verifiable. For instance, the government can give away in period 1 options that will enable the agents to buy pollution permits at price $p_2^*(\kappa)$ (see subsection 3.2 for its definition) in state of nature κ. Alternatively the government could commit to a state-contingent price support policy.

Time consistency is harder to achieve when the state of nature is not verifiable. Suppose that κ is learned by the government only. The government may then be tempted to understate κ, that is to understate the pollution damage in order to flood the market with new permits. One may then think about price support policies that restore incentive compatibility for the government. Suppose that in period 1 the government sells $\bar{n}_2 \equiv n_2^*(\bar{\kappa})$ futures permits, where $\bar{\kappa}$ corresponds to the highest possible pollution damage. Let the government set a price $\bar{p}_2$ and commit to a refund policy $r(p_2)$ per permit for any price $p_2 \leq \bar{p}_2$, with $r(\bar{p}_2) = 0$ say. (The $\bar{n}_2$ futures permits are sold at a price that clears the market given the refund policy and the fact that, in equilibrium the government must want to sell new permits at a market clearing price $p_2^*(\kappa)$ in each state of nature κ.) Can a refund policy implement the commitment outcome? Using the notation of subsection 3.2, in state of nature κ, the government chooses p_2 so as to maximize:

$$W_2(p_2, \theta^*, \kappa) - \lambda[r(p_2) + p_2]\bar{n}_2.$$

The analysis in subsection 3.2 suggests choosing $r(\cdot)$ such that

$$\lambda[r'(p_2) + 1]\bar{n}_2 = \mu[1 - G_2(p_2|\theta^*)]$$

or

$$r'(p_2) = \frac{\mu}{\lambda \bar{n}_2}[1 - G_2(p_2|\theta^*)] - 1.$$

Recall that in the absence of aggregate uncertainty, the price support rule satisfied $r'(p_2) = -1$. Here it is concave in p_2, because a second-period price reduction deters investment more in high- than in low- pollution states. Last, for the price support policy $r(\cdot)$ just obtained to yield the commitment outcome it must be the case that the government's second-period objective function be concave. Because $(-r(p_2))$ is convex, W_2 must be sufficiently concave in p_2 (say, because D_2 is very convex.)

5 Comparing markets and optimal pollution control: overall regulation

We saw that the introduction of a futures market reduces inefficient bypass and improves on a setting of sequential spot markets. Yet, and unlike in the static case, markets do not yield optimal pollution control. It may be useful to give the intuition why markets are not optimal. The second-period price p_2 plays several roles: it determines the second-period pollution level, it allocates (in an ex post efficient way) the pollution among agents, and it guides investment. Clearly, it would be useful to have another instrument to control investment. Perhaps less obviously, more instruments can also help extract the agents' rent.

To see why a futures price p_2 is a poor rent extraction instrument, let us first recall that it corresponds to a free option with exercise price p_2. At the very least the government could sell this option at price $R_0 = \delta E[\max(\theta_2 - p_2, 0)|0] > 0$. All agents would still buy the option and welfare would increase by λR_0.[17] Still, charging for an option with a uniform striking price is not optimal. The government can exploit the temporal correlation of willingnesses to pay by practicing a form of (intertemporal) nonlinear pricing and offering a menu of options. Agents with a high first-period valuation reveal their type by demanding an expensive option with a low striking price; because they are likely to have a high willingness to pay in period 2 and therefore to use a permit (if they have not invested), they are eager to pay a low striking price.

Section 6 will derive the optimal (commitment) mechanism under the assumption that the environmental regulator observes only the agents' pollution level, and not their investment or production. This case corresponds to *pure environmental regulation.* Pure environmental regulation is directly comparable with the market system studied in previous sections, because both posit that pollution levels are the only control instruments. This section analyzes the simpler case of *overall regulation* in which the regulator can observe the agents' pollution level, investment and production.[18] We first present the abstract mechanism design problem and then give a simple interpretation of our findings.

[17] Actually, it is clear that by decreasing p_2 slightly and raising the price of the option R from R_0 to $R_0 + \delta R$ such that type $\theta_1 = 0$ is still indifferent between buying the option and not buying it, one increases welfare by $\lambda \delta R$ because the change in p_2 around the optimal commitment price has only a second-order effect.

[18] In our model, the observation of period-2 production and pollution would give the relevant information about investment. So, one might as well assume that both or none of the two are observed. The overall regulation case is simpler to analyze because the investment choice does not introduce an extra incentive constraint.

5.1 Mechanism design

At an abstract level, the government sets up at date 1 a mechanism, in which each agent makes announcements $\hat{\theta}_1$ and $\hat{\theta}_2$ of his first- and second-period valuations, in periods 1 and 2 respectively. The agent's second-period announcement is contingent on whether investment actually occurred. The second-period announcement can thus be denoted $\hat{\theta}_2^i$ (investment took place) or $\hat{\theta}_2^{ni}$ (the agent did not invest). The *mechanism* specifies a probability $x_1(\hat{\theta}_1)$ of being allowed to produce/pollute in period 1, a probability $y(\hat{\theta}_1)$ of investing, probabilities $x_2^i(\hat{\theta}_1, \hat{\theta}_2^i)$ and $x_2^{ni}(\hat{\theta}_1, \hat{\theta}_2^{ni})$ of being allowed to produce in period 2 and transfers $T_2^i(\hat{\theta}_1, \hat{\theta}_2^i)$ and $T_2^{ni}(\hat{\theta}_1, \hat{\theta}_2^{ni})$ from the agent to the government, where the superscripts "i" and "ni" refer to the fact that the agent invested in period 1 or not. We adopt the convention that transfers are made in period 2.

a) Agents' behavior. Let $U(\theta_1)$ denote the utility or rent of an agent with type θ_1:

$$\begin{aligned}
U(\theta_1) &= \max_{\{\hat{\theta}_1, \hat{\theta}_2^i, \hat{\theta}_2^{ni}\}} \{\theta_1 x_1(\hat{\theta}_1) + y(\hat{\theta}_1)[-i + \delta \textstyle\int_0^1 [\theta_2 x_2^i(\hat{\theta}_1, \hat{\theta}_2^i) \\
&- T_2^i(\hat{\theta}_1, \hat{\theta}_2^i)] g_2(\theta_2|\theta_1) d\theta_2] + (1 - y(\hat{\theta}_1))\delta[\textstyle\int_0^1 [\theta_2 x_2^{ni}(\hat{\theta}_1, \hat{\theta}_2^{ni}) \\
&- T_2^{ni}(\hat{\theta}_1, \hat{\theta}_2^{ni})] g_2(\theta_2|\theta_1) d\theta_2]\}
\end{aligned}$$

where the first-period announcement is contingent on θ_1, and the second-period announcement is contingent on θ_1, on the realization of the investment and on θ_2. The revelation principle implies that there is no loss of generality in focusing on mechanisms that induce the agent to reveal truthfully. Using the envelope theorem and an integration by parts, the slope of the rent function is given by:[19]

$$\begin{aligned}
\dot{U}(\theta_1) &= x_1(\theta_1) - \delta[y(\theta_1) \int_0^1 \frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1) x_2^i(\theta_1, \theta_2) d\theta_2 \\
&+ (1 - y(\theta_1)) \int_0^1 \frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1) x_2^{ni}(\theta_1, \theta_2) d\theta_2].
\end{aligned} \tag{8}$$

The agent is willing to participate in the program only if $U(\theta_1) \geq 0$ for all θ_1. Because the agent's rent increases with his type (see equation (8)), the individual rationality

[19] The second-order conditions of incentive compatibility of this problem are complex. To guarantee truthful revelation of θ_2 in period 2 it is necessary and sufficient that $x_2^{ni}(\theta_1, \theta_2)$ and $x_2^i(\theta_1, \theta_2)$ be nondecreasing in θ_2. This obtains if

$$\theta_2 \longrightarrow \theta_2 - \frac{\lambda}{1+\lambda} \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \frac{\left(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)\right)}{g_2(\theta_2|\theta_1)}$$

is increasing in θ_2 for all θ_1 (which is the case for instance if λ is not too large) and nondecreasing in θ_1 for all θ_2 as well (which holds if $\frac{1-F_1(\theta_1)}{f_1(\theta_1)}$ is nonincreasing in θ_1 and $\frac{\frac{\partial G_2}{\partial \theta_1}}{g_2}$ nonincreasing in θ_1). The second-order conditions of incentive compatibility in period 1 are complex and are dealt with in Appendix 2.

constraint can be written as:

$$U(0) = 0. \tag{9}$$

The agents' total rent is therefore:

$$\int_0^1 U(\theta_1) f_1(\theta_1) d\theta_1 = \int_0^1 \dot{U}(\theta_1)[1 - F_1(\theta_1)] d\theta_1.$$

b) Welfare maximization. The government's optimization program is:

$$\max_{\{x_1(\cdot), y(\cdot), x_2^i(\cdot,\cdot), x_2^{ni}(\cdot,\cdot)\}} \{-D_1(\textstyle\int_0^1 x_1(\theta_1) f_1(\theta_1) d\theta_1)$$

$$-\delta D_2(\textstyle\int_0^1 [1 - y(\theta_1)][\int_0^1 x_2^{ni}(\theta_1, \theta_2) g_2(\theta_2|\theta_1) d\theta_2] f_1(\theta_1) d\theta_1)$$

$$+(1+\lambda) \textstyle\int_0^1 [\theta_1 x_1(\theta_1) + y(\theta_1)(-i + \delta \int_0^1 \theta_2 x_2^i(\theta_1, \theta_2) g_2(\theta_2|\theta_1) d\theta_2)$$

$$+(1 - y(\theta_1))\delta \textstyle\int_0^1 \theta_2 x_2^{ni}(\theta_1, \theta_2) g_2(\theta_2|\theta_1) d\theta_2] f_1(\theta_1) d\theta_1$$

$$-\lambda \textstyle\int_0^1 \dot{U}(\theta_1)[1 - F_1(\theta_1)] d\theta_1)\},$$

where $\dot{U}(\theta_1)$ is given by equation (8). Note that social welfare is divided into three components: the social cost of pollution damage in both periods, the expected value of production (weighted at $1 + \lambda$, because it would be equal to the monetary transfer received by the government if the latter could fully capture the agents' rents), and the cost of leaving rents to the agents (proportional to λ.) Let $d_1 = D_1'(n_1)$ and $d_2 = D_2'(n_2)$ denote the social marginal costs of pollution. Pointwise maximization of social welfare yields the following first-order conditions.

The maximization with respect to $x_1(\theta_1)$ gives the now standard monopoly formula (a rewriting of equation (1·)):

$$\begin{aligned} x_1(\theta_1) &= 1 \quad \text{if} \quad \theta_1 \geq \frac{d_1}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{10}$$

Let

$$p_1^* = \frac{d_1}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{1 - F_1(p_1^*)}{f_1(p_1^*)}. \tag{11}$$

For types who do not invest, the maximization over $x_2^{ni}(\theta_1, \theta_2)$ yields:

$$\begin{aligned} x_2^{ni}(\theta_1, \theta_2) &= 1 \quad \text{if} \quad \theta_2 \geq \frac{d_2}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \frac{(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1))}{g_2(\theta_2|\theta_1)} \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{12}$$

We assume that the function $\theta_2 \longrightarrow \theta_2 - \frac{\lambda}{1+\lambda} \frac{1-F_1(\theta_1)}{f_1(\theta_1)} \frac{(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1))}{g_2(\theta_2|\theta_1)}$ is increasing in θ_2 for all θ_1 (which is the case for instance if λ is not too large), and increasing in θ_1 as well. [These conditions will be used to ensure that agents' second-period second-order conditions are satisfied.] We can then define a single-valued nonincreasing function $p_2(\cdot)$ by:

$$p_2(\theta_1) \equiv \frac{d_2}{1+\lambda} + \frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\frac{(-\frac{\partial G_2}{\partial \theta_1}(p_2(\theta_1)|\theta_1))}{g_2(p_2(\theta_1)|\theta_1)}. \tag{13}$$

For types who do invest, the maximization over $x_2^i(\theta_1, \theta_2)$ yields the same condition as under no investment, except that production no longer entails a pollution cost:

$$\begin{aligned} x_2^i(\theta_1,\theta_2) &= 1 \quad \text{if} \quad \theta_2 \geq \frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\frac{(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1))}{g_2(\theta_2|\theta_1)} \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{14}$$

We define a new, nonincreasing function $q_2(\cdot)$ by:

$$q_2(\theta_1) \equiv \frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\frac{(-\frac{\partial G_2}{\partial \theta_1}(q_2(\theta_1)|\theta_1))}{g_2(q_2(\theta_1)|\theta_1)} < p_2(\theta_1). \tag{15}$$

Last, we optimize with respect to the investment decision $y(\theta_1)$,using the fact that the agent produces in period 2 if and only if $\theta_2 \geq p_2(\theta_1)$ (in the absence of investment) or $\theta_2 \geq q_2(\theta_1)$ (if the agent has invested):

$$\begin{aligned} y(\theta_1) &= 1 \quad \text{if} \quad -i + \delta\Big[\int_{q_2(\theta_1)}^{p_2(\theta_1)} \theta_2 g_2(\theta_2|\theta_1)d\theta_2 + \frac{d_2}{1+\lambda}(1-G_2(p_2(\theta_1)|\theta_1))\Big] \\ &\quad +\delta\frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\int_{q_2(\theta_1)}^{p_2(\theta_1)}\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)d\theta_2 \geq 0, \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{16}$$

We assume that the left-hand side of the condition in (16) is increasing in θ_1 (which is the case in particular if λ is not too large.) Then there exists a cutoff θ^* such that an agent invests if and only if his type exceeds θ^*. The reader will check that the cutoff exceeds its first best level. That is, the regulator restricts investment in order to better extract the agents' rent.[20]

Note that condition in (16) has a simple interpretation. The investment has one benefit and one cost (besides the monetary investment cost i). It eliminates the pollution of those types $\theta_2 \geq p_2(\theta_1)$ that polluted; and it allows production by types in $[q_2(\theta_1), p_2(\theta_1)]$. On the other hand, an agent's rent grows faster with θ_1 in case of investment because second-period production becomes more likely.

We still need to check that the global second-order condition for an agent's maximization program is satisfied. We will do so when we describe the implementation.

Last, let $I(\theta_2|\theta_1) = -\frac{\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)}{g_2(\theta_2|\theta_1)}$ denote the informativeness of θ_1 about the second-period needs for pollution rights. As the informativeness of θ_1 goes uniformely to zero for

[20]See appendix 1

every θ_1 (the types are intertemporally independent in the limit), the optimal mechanism converges to a mechanism which can be implemented by a sequence of spot markets with a supply of pollution rights determined each period by the regulator. Furthermore, either all firms should invest or none.

When the uncertainty about future needs is large it is intuitively clear that the ex post efficiency of period 2 spot markets is very attractive. Alternatively one can study the design of policies when the existence of spot markets each period is a constraint. This constraint implies ex post productive efficiency conditionally on the number of pollution rights. It excludes the price discrimination involved in the optimal mechanism. Consequently we can expect too much rent to be given up to firms in period 2. When investment is controlled by the regulator, investment will be decreased in period 1 to induce less (costly) production in period 2. When investment is not controllable, investment can be similarly decreased indirectly by committing to a larger (than ex post optimum) level of pollution rights (and a smaller price of pollution rights). See section 6.

5.2 Implementation

In the optimal mechanism the regulator uses the correlation of types across periods to decrease the cost of asymmetric information by committing to period-two allocations which are not ex post efficient (since $p_2(.)$ and $q_2(.)$ depend on the first-period type). In contrast markets yield ex post efficient allocations and therefore cannot implement the optimum. The optimal mechanism price-discriminates for the period-two rights to produce and to pollute on the basis of the first-period type.

In period 1, the optimum can be implemented by a market for first-period pollution permits at a uniform price p_1^* given by (11) and a set of options markets : For the agents who commit not to invest, the regulator offers a menu of options $\{p_2, R(p_2\}$, where p_2 is the *striking price* to be paid in period 2 for *polluting* in period 2 and $R(p_2)$ is the purchase price (in period 1) of an option with exercice price p_2 (where $R(.)$ is a decreasing function defined below). For the agents who commit to invest the regulator offers a second menu of options $\{q_2, S(q_2)\}$, where q_2 is the striking price or *usage tax* to be paid for *producing* in period 2 and $S(q_2)$ is the purchase price of an option with exercice price q_2 (where $S(.)$ is a decreasing function).

Agents with higher θ_1 value know that (conditionally on not investing) they are more likely to need a pollution permit tomorrow, and buy better (lower striking price) options today. Similarly, agents with a higher willingness to pay in period 1 buy lower usage taxes.

The purchase price of options to pollute is computed so that an agent who does not invest (namely, with type $\theta_1 < \theta^*$) picks the option with exercice price $p_2(\theta_1)$ defined by (13). Let us compute the first-period price $R(p_2)$ of an option with striking price p_2 and that induces this choice. Assuming that the first-period allocation is implemented by the spot market of pollution permits with price p_1^*, the agent's utility level in period 2 (discounted at period 1) is, for a type-θ_1 agent claiming he has type $\tilde{\theta}_1$:

$$U_2(\theta_1, \tilde{\theta}_1) \equiv -R(p_2(\tilde{\theta}_1)) + \delta \int_{p_2(\tilde{\theta}_1)}^{1} (\theta_2 - p_2(\tilde{\theta}_1)) g_2(\theta_2|\theta_1) d\theta_2.$$

The first-order condition of incentive compatibility is :

$$\Big[- R'(p_2(\theta_1)) - \delta(1 - G_2(p_2(\theta_1)|\theta_1)) \Big] p_2'(\theta_1) = 0.$$

Or, if we define $\theta_1(p_2)$ as the inverse function of $p_2(\theta_1)$ defined by (13),

$$R'(p_2) = -\delta\Big(1 - G_2(p_2|\theta_1(p_2))\Big) < 0. \tag{17}$$

Note that the second-order condition of incentive compatibility amounts to

$$\delta\Big(\frac{\partial G_2}{\partial \theta_1}(p_2(\theta_1)|\theta_1)\Big) p_2'(\theta_1) \geq 0$$

or

$$p_2'(\theta_1) \leq 0.$$

The function $R(.)$ is obtained from (17) and the binding participation constraint $U(0,0) = 0$.

Similarly, the "second-period" utility of those who commit to invest (that is, who have type $\theta_1 > \theta^*$) is :

$$V_2(\theta_1, \tilde{\theta}_1) = -S(q_2(\tilde{\theta}_1)) - i + \delta \int_{q_2(\tilde{\theta}_1)}^{1} (\theta_2 - q_2(\tilde{\theta}_1)) g_2(\theta_2|\theta_1) d\theta_2$$

leading to

$$S'(q_2) = -\delta\big[1 - G_2(q_2|\theta_1(q_2))\big] \text{ with } q_2(\theta_1(q_2)) = q_2$$

with the "pasting condition"

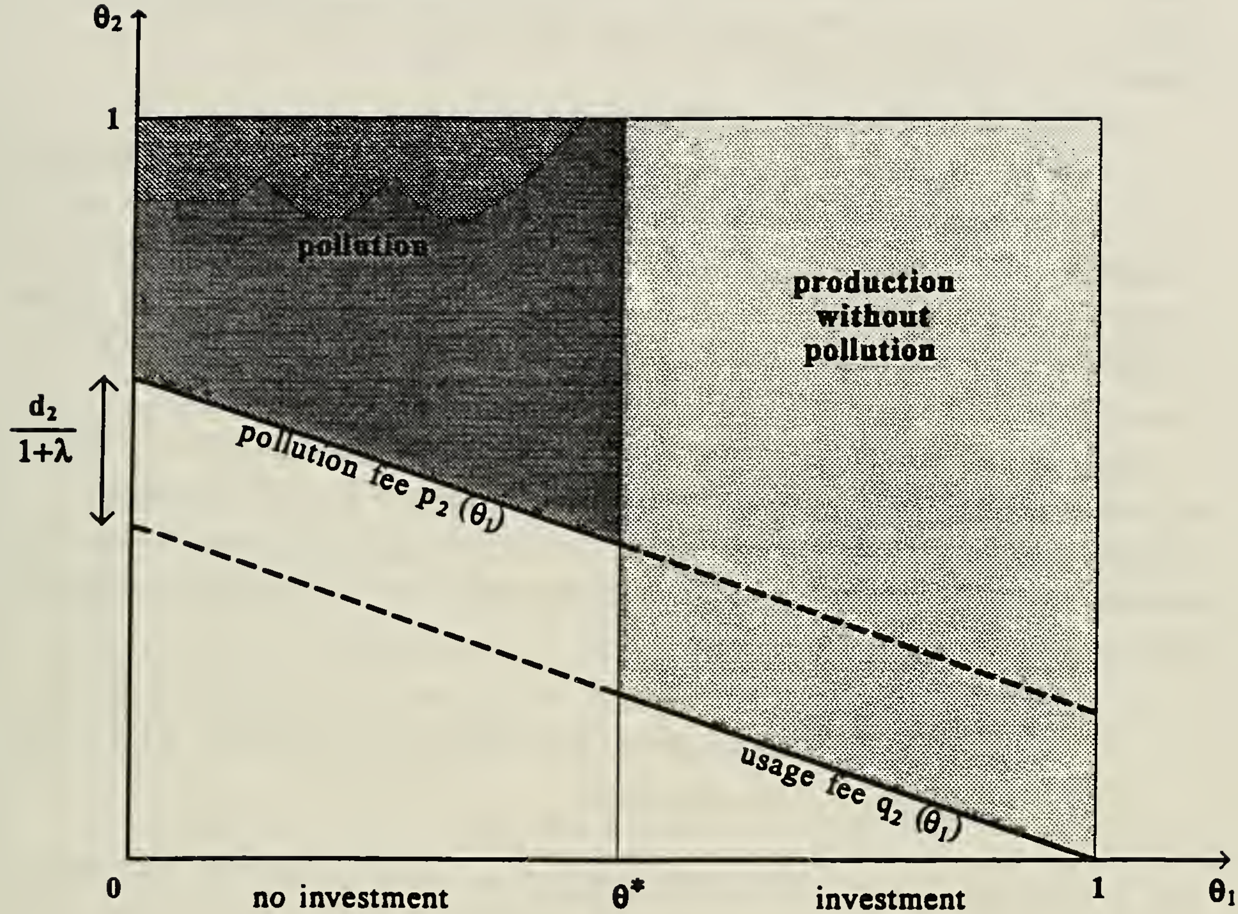
$$V(\theta^*, \theta^*) = U(\theta^*, \theta^*).$$

The optimal mechanism is illustrated in Figure 1.

FIGURE 1 HERE

Let us reemphasize that pollution permits of period 2 cannot be traded as agents with different first-period valuations must face different second-period striking prices and therefore the second-period allocation cannot be efficient.

We summarize our analysis in:



θ2
1
pollution
production
without
pollution
d2
1+λ
pollution fee p2 (θ1)
usage fee q2 (θ1)
0
no investment
θ*
investment
1
θ1

Figure 1

Proposition 3 *Under overall regulation, the government offers two menus of option contracts depending on whether the agent chooses to invest. In each menu, a higher-valuation agent chooses a more expensive option with a lower striking price. The allocation of pollution is not ex post efficient. Last, the government taxes investment and induces less investment than in the first best.*

d) Remark : In this section we have derived the benchmark of an optimal mechanism when pollution and investment are controllable and when the regulator can commit intertemporally. To analyze the performance of markets for pollution rights, it would be useful to have three other benchmarks, namely the case where the regulator still controls investment but cannot commit, the case where he cannot control investment and can commit, and finally the case where both problems occur.

Unfortunately the analysis of limited commitment or renegotiation proof commitment is quite complex because of the ratchet effect (see chapters 9 and 10 in Laffont-Tirole (1993)). A thorough analysis in the case of a continuum of types like here is beyond the scope of this paper. In section 6, we discuss the optimal mechanism when investment is unobservable.

6 Comparing markets and optimal control: pure environmental regulation

We now assume that the federal government (or one of its agencies) can only regulate pollution and has no control over investment and production. Unlike in section ?? the government then cannot levy a tax on production nor can it directly control (or tax) investment; it can (and will) still issue nontradeable options on pollution permits, but these options must now play the triple role of allocating pollution rights, guiding investment and extracting the agents' rents. A complete analysis of pure environmental regulation lies outside the scope of this paper. We content ourselves with making a few points concerning this situation. We set up the incentive problem and then look at two polar cases.

The government sets up at the beginning of date 1 a mechanism in which each agent makes announcements $\hat{\theta}_1$ and $\hat{\theta}_2$ of his first- and second-period valuations, in periods 1 and 2 respectively. Without loss of generality, the mechanism elicits the agents' information truthfully. It also splits the agents into two groups. Those in the subset of types Θ_1 invest. For types $\theta_1 \in \Theta_1$, the mechanism specifies a probability $x_1(\theta_1)$ of being allowed to pollute in period 1 and a transfer $t(\theta_1)$ to the regulator. For types $\theta_1 \in \Theta_1^c$ (the complement of Θ_1), who do not invest, the mechanism specifies probabilities $x_1(\theta_1)$ and

$x_2(\theta_1, \theta_2)$ of being able to pollute in periods 1 and 2, respectively, and a transfer $T(\theta_1, \theta_2)$ to the regulator. We adopt the convention that these transfers are date-2 transfers.[21]

The rent $U(\theta_1)$ of an agent with type θ_1 is:

$$\begin{aligned} U(\theta_1) \quad = \quad \max\{ \quad & \max_{\hat{\theta}_1 \in \Theta_1}[x_1(\hat{\theta}_1)\theta_1 - \delta t(\hat{\theta}_1)] + \max[0, -i + \delta E(\theta_2|\theta_1)], \\ & \max_{\hat{\theta}_1 \in \Theta_1^c}[x_1(\hat{\theta}_1)\theta_1 + \max[E(\max_{\hat{\theta}_2} \delta x_2(\hat{\theta}_1, \hat{\theta}_2)\theta_2 - \delta T(\hat{\theta}_1, \hat{\theta}_2)|\theta_1), \\ & \delta E(\theta_2|\theta_1) - i - \delta \min_{\hat{\theta}_2} T(\hat{\theta}_1, \hat{\theta}_2)]\}. \end{aligned} \tag{18}$$

The first strategy for the agent is to apply (or not) for a date-1 pollution right only. Even though *on the equilibrium path*, agents who apply for a period 1 pollution right only do invest (wlog, since not investing and not producing in period 2 is equivalent to not investing and having an option at a striking price above 1), we must consider the possibility that the agent elects not to invest.

The second strategy consists in claiming one is not investing ($\hat{\theta}_1 \in \Theta_1^c$), and thus be entitled to choosing in $\{x_2(\hat{\theta}_1, \cdot), T(\hat{\theta}_1, \cdot)\}$ in period 2. Note that the agent can claim he is not investing and actually invest. Let $\theta^\dagger$ be defined by $\delta E(\theta_2|\theta^\dagger) = i$. Using the envelope theorem and integrating by parts, the gradient of the rent is as usual equal to the expected present discounted amount of production:

$$\dot{U}(\theta_1) \quad = \quad \begin{cases} x_1(\theta_1) \quad \text{for} \quad \theta_1 \in \Theta_1 \cap [0, \theta^\dagger], \\ x_1(\theta_1) + \delta \frac{d}{d\theta_1} E(\theta_2|\theta_1) \quad \text{for} \quad \theta_1 \in \Theta_1 \cap [\theta^\dagger, 1], \\ x_1^c(\theta_1) + \delta \int_0^1 (-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)) x_2(\theta_1, \theta_2) d\theta_2 \quad \text{for} \quad \theta_1 \in \Theta_1^c. \end{cases} \tag{19}$$

Last, the agent's individual rationality constraint can be written

$$U(\theta_1) \geq \max(0, \delta E(\theta_2|\theta_1) - i). \tag{20}$$

The difficulty of the problem stems not so much from the apparent complexity of the incentive compatibility condition (18), which can be replaced by condition (19) together with an ex post verification of global optimality for the agent. Rather, the individual rationality condition (20) creates trouble, as it may a priori bind at different values in the interval $[0, 1]$. As is well known the analysis of type -dependent reservation utilities is complex.

[21]The reader might be concerned that the regulator would find out in the following way whether the agent invested. An agent who invests and announces he has not invested could be asked to pollute at least with some probability. To rule this out, we assume that the agent can always discard the pollution-abatement investment and behave like an agent who has not invested. Conversely, can an agent who does not invest disguise as one who invests? The answer is yes since the agent can always elect not to produce and therefore not pollute. This strategy of course may have a cost, which will be formalized when we look at the incentive compatibility condition on investment.

a) In order to disentangle some effects of the non observability of investment and production from those of an individual rationality constraint binding above the lowest valuation, we first look at the case in which (20) takes the more familiar form :

$$U(\theta_1) \geq 0 \text{ for all } \theta_1.$$

As we will see, we can ignore the bypass option as far as individual rationality is concerned, as long as the high valuation types value first-period production enough (that is, if the optimal price for first-period permits is low enough) that they would not contemplate refusing to interact with the pollution regulator.

We guess the solution by ignoring some of the constraints that are implicit in (18) and (19). The ignored constraints will be checked ex post. Note first that (19) implies that for all θ_1

$$\dot{U}(\theta_1) = x_1(\theta_1) + \delta \int_0^1 \Big(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)\Big) x_1(\theta_1, \theta_2) d\theta_2 \tag{21}$$

as x_2 can take any value between 0 and 1. (So, $x_2(\theta_1, .) \equiv 0$ for $\theta_1 \in \Theta_1 \cap [0, \theta^\dagger]$ and $x_2(\theta_1, .) \equiv 1$ for $\theta_1 \in \Theta_1 \cap [\theta^\dagger, 1]$). While (21) implies neither (18) nor (19), we use (21) in our optimization program. Positing for the moment that the individual rationality constraint is binding only at $\theta_1 = 0$ (which must also be checked ex post), we also have :

$$U(\theta_1) = \int_0^{\theta_1} \dot{U}(\theta) d\theta.$$

We can now build on the mechanism design of section 5. The objective function, the incentive constraint and the individual rationality constraint are the same as in section 5. The only difference is that the regulator has fewer instruments. Namely, an environmental regulator does not control $y(\theta_1)$ directly, and cannot prevent second-period production if the agent invests. So, we necessarily have :

$$x_2^i(\theta_1, .) \equiv 1 \Leftrightarrow q_2(\theta_1) = 0. \tag{22}$$

What are the implications of the lack of control aver investment ? Let us consider the fictitious case in which the regulator can control investment but is constrained to $q_2(\theta_1) = 0$. Equation (16), when modified, yields the optimal cutoff $\theta^\otimes$:

$$\begin{aligned} y(\theta_1) = 1 \quad \text{if} \quad & -i + \delta \Big[\int_0^{p_2(\theta_1)} \theta_2 g_2(\theta_2|\theta_1) d\theta_2 + \frac{d_2}{1+\lambda}\big(1 - G_2(p_2(\theta_1)|\theta_1)\big)\Big] \\ & + \delta \frac{\lambda}{1+\lambda} \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \int_0^{p_2(\theta_1)} \frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1) d\theta_2 \geq 0 \\ & = 0 \text{ otherwise.} \end{aligned} \tag{23}$$

Note that $\theta^\otimes > \theta^*$: the fact that the regulator loses control over production when investment occurs makes investment socially less desirable.

We must now look for conditions under which the environmental regulator can de facto control investment even though he has no statutory power to do so. For expositional purposes, it is convenient to move into the *implementation* of the optimal regulation (which follows the lines of section 5.2).

The regulator sells first-period pollution permits at price p_1^* to agents who simultaneously buy second-period options. That is, the environmental regulator *bundles* some first -and second-period rights. The second-period options $\{p_2, R(p_2)\}$ satisfy, as in section 5.2. :

$$R'(p_2(\theta_1)) = -\delta\big[1 - G_2(p_2(\theta_1)|\theta_1))\big].$$

Now, let us assume that $p_1^* < \theta^{\otimes}$ (tied first-period pollution rights are cheap). Let the regulator sell *unbundled* first-period rights at price $p_1^{\otimes}$ such that :

$$\theta^{\otimes} - p_1^* - R(p_2(\theta^{\otimes})) + \delta \int_{p_2(\theta^{\otimes})}^{1} (\theta_2 - p_2(\theta^{\otimes}))g_2(\theta_2|\theta^{\otimes})d\theta_2 = \theta^{\otimes} - p_1^{\otimes} - i + \delta E(\theta_2|\theta^{\otimes}). \quad (24)$$

$$p_1^{\otimes} > p_1^*. \quad (25)$$

It is easily shown that if $p_1^{\otimes} \leq \theta^{\otimes}$, no agent wants to bypass without buying a first-period pollution permit, and therefore the individual rationality constraint takes the familiar form $U(\theta_1) \geq 0$ for all θ_1. Provided that $p_1^{\otimes} \leq \theta^{\otimes}$, Appendix 3 proves our main result :

The bundled first-period permit is cheaper that its unbundled counterpart.

b) We have examined one polar case in which the regulator can control the level of investment by tying the purchase of second-period pollution rights to the first-period price. Controlling investment is then costless because agents have a large stake in period 1.

The other extreme case we can briefly consider is when the pollution costs are so high in period 1 that noboby purchases permits. The role of the IR contraint is then quite transparent.

Suppose (without loss of generality for this reasoning) that $\delta E(\theta_2|0) - i = 0$. Then if $U(0) = 0$, type 0, and actually all types bypass.[22] The only way to avoid complete bypass is to leave a rent to those who purchase pollution rights.

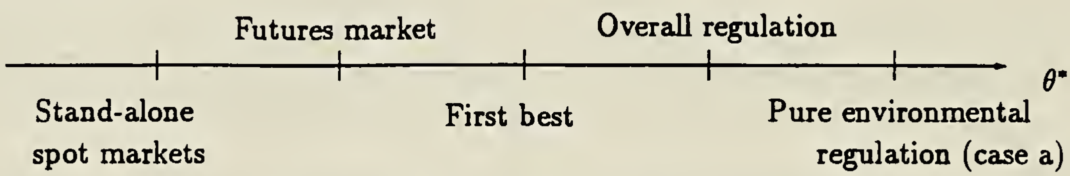
Optimizing social welfare respect to $\theta^{\otimes}$ and $p_2(\theta_1)$ we obtain the two following results (see Appendix 4). There is *more bypass* that in the case where the marginal firm $\theta^{\otimes}$ can be controlled through the purchase of a first-period pollution permit (case a). To

[22]This results from the slope of U under bypass weakly exceeding that under any permit system.

counteract this excessive bypass, the striking prices of options offered in period one are *below marginal cost.*[23]

7 Desirable extensions

We can summarize the way investments in private pollution abatement technologies are affected by policy instruments with the following graph giving the marginal agent engaging in the bypass activity.



Stand-alone spot markets :	The regulator does not observe investments. His only instrument is the supply of pollution rights each period.
Futures market :	The regulator does not observe investments. He can offer in period 1 pollution rights for period 2 with the commitment of not changing the supply of pollution rights in period 2.
First best :	The regulator observes investments and has complete information about the agents'.
Overall regulation :	The regulator observes investments but has incomplete information about the agents' valuations. He can control the supply of pollution rights, can offer in period 1 menus of options of pollution rights in period 2 and can make transfers.
Pure environmental regulation :	The regulator does not observe investments and has incomplete information about the agents' valuations. The regulator only observes pollution, but, unlike under "futures market", can offer nonassignable, contingent permits.

We conclude with a (non-exhaustive) list of desirable extensions. First, in our model the futures market guides investment by constraining the future spot price. In practice, the futures market also supplies information about this spot price. One could formalize this "price discovery" by allowing the distribution of first-period valuations in the population of agents to be ex ante uncertain to the regulator. Second, we have assumed that the agents' investment cost is known. We could more generally allow the investment cost

[23]Laffont-Tirole (1990) has shown how nonlinear prices used to fight bypass may entail marginal prices lower than marginal costs.

to differ among agents, and also be subject to moral hazard.[24] This extension would be particularly relevant for the SO_2 market, in which about seventy percent of the US pollution comes from public utilities. Third, overall regulation, when it prevails, need not be performed by a single regulator, but rather (as is the case for US electric utilities) by several agencies with conflicting objectives. Fourth, we have assumed that the government has the means to achieve its social goals. In practice, the government may have to obtain political support from interest groups (this motivation is sometimes offered to defend grandfathering.) Analyses of support-building for reform (Dewatripont-Roland (1992)) might be relevant here. We hope that future research will develop these (and other) interesting extensions.

[24] As in Laffont-Tirole (1993, p86–103).

References

Baron,D. and D. Besanko [1984] "Regulation and Information in a Continuing Relationship", *Information Economics and Policy*, 1:447–470.

Burkart, M. [1993] "Action and Auction: The SO_2 Emissions Allowances", *Public Utilities Fortnightly*, June 15, 42–44. 1:447–470.

Costello, K. [1992] "Comments on: 'Demand-Side Management: Reflections of an Irreverent Regulator' by Myron Katz", *Resources and Energy*, 14:205–214.

Dewatripont, M. and G. Roland [1992], "Economic Reform and Dynamic Political Constraints", *Review of Economic Studies*, 59: 703–730.

Fudenberg, D. and J. Tirole [1991] *Game Theory*, Cambridge: MIT Press.

Hahn, R. and R. Noll [1983], "Barriers to Implementing Tradeable Air Pollution Permits: Problems of Regulatory Interactions", *Yale Journal of Regulation*, 1: 63–91.

Hausker, K. [1992], "The Politics and Economics of Auction Design in the Market for Sulfur Dioxide Pollution", *Journal of Policy Analysis and Management*, 11:553–572.

Laffont, J.-J. and J. Tirole [1990] "Bypass and Creamskimming", *American Economic Review*, 80:1042–1061.

Laffont, J.-J. and J. Tirole [1993] *A Theory of Incentives in Procurement and Regulation*, Cambridge: MIT Press.

Laffont, J.-J. and J. Tirole [1994a] "Environmental Policy, Compliance and Innovation ", *European Economic Review Papers and Proceedings*, 38:555–562.

Laffont, J.-J. and J. Tirole [1994b] " A Note on Environmental Innovation ", Fondazione Eni Enrico Mattei, discussion paper.

NRRI (National Regulatory Research Institute) [1992] "Public Utility Commission Implementation of the Clean Air Act's Allowance Trading Program" (NRRI, Columbus, OH).

Public Utilities Report [1993] *The New Clean Air Act: Compliance and Opportunity*, edited by R. Lock and D. Harkawik (Arlington, VA).

Rose, K. and R. Burns [1993] "Regulatory Policy issues and the Clean Air Act", mimeo (National Regulatory Research Institute, Columbus, OH).

Tietenberg, T.H. [1985] *Emissions Trading*, Resources for the Future, Washington D.C.

Wessler,E. [1992] Discussion Paper on Wholesale Ratemaking Considerations for Sulfur Dioxyde Emissions Allowance Trading, mimeo (FERC, Washington, DC).

Appendix 1 : Proof that asymmetric information decreases bypass

Condition (16) differs from the first best condition in that "$q_2(\theta_1)$" replaces "0", "$p_2(\theta_1)$" replaces "$\frac{d_2}{1+\lambda}$" as second-period cutoff, and the last term in (16) is added. To show that the incentive to invest is lower than in the first best start from the first best condition

$$-i + \delta \int_0^{\frac{d_2}{1+\lambda}} \theta_2 g_2 d\theta_2 + \frac{d_2}{1+\lambda}(1 - G(\frac{d_2}{1+\lambda})) = 0$$

Add the negative term $\delta \frac{\lambda}{1+\lambda} \frac{1-F_1}{f_1} \int_{q_2(\theta_1)}^{\frac{d_2}{1+\lambda}} \frac{\partial G_2}{\partial \theta_1} d\theta_1$

and change variables to obtain

$$-i + \delta \int_{q_2}^{p_2} \theta_2 g_2 d\theta_2 + \frac{d_2}{1+\lambda}(1 - G(p_2)) + \delta \frac{\lambda}{1+\lambda} \frac{1-F_1}{f_1} \int_{q_2(\theta_1)}^{p_2} \frac{\partial G_2}{\partial \theta_1} d\theta_2. \qquad (A1)$$

At $q_2 = 0$ and $p_2 = \frac{d_2}{1+\lambda}$ this expression is negative.

Take the derivatives with respect to p_2 and q_2. We obtain

$$p_2 g_2(p_2) - \frac{d_2}{1+\lambda} g_2(q_2) + \delta \frac{\lambda}{1+\lambda} \frac{1-F_1}{f_1} \frac{\partial G_2}{\partial \theta_1}(p_2) \qquad (A2)$$

and

$$-\delta q_2 g_2(q_2) < 0.$$

From the second order condition of footnote 14 and the definition of $p_2(\theta_1)$, (A2) is negative. Therefore the expression (A1) with $p_2 = p_2(\theta_1)$ and $q_2 = q_2(\theta_1)$ is negative. Since (A1) is increasing in θ_1 the cutoff value is higher.

Q.E.D.

Appendix 2 : Second-order conditions of the agents' first-period optimisation problem (section 5).

To study the second-order conditions of the agents' first-period optimization problem, it is convenient to use the implementation developed in section 5.2. Let

$$\hat{R}(\theta_1) \equiv \begin{cases} R(p_2(\theta_1)) & \text{if } \theta_1 < \theta^* \\ K & \text{if } \theta_1 \geq \theta^* \end{cases}$$

$$\hat{p}_2(\theta_1) \equiv \begin{cases} p_2(\theta_1) & \text{if } \theta_1 < \theta^* \\ q_2(\theta_1) & \text{if } \theta_1 \geq \theta^*. \end{cases}$$

$K > R(p_2(\theta^*))$ is chosen so as to make type θ^* indifferent between investing and not investing.

The agent's optimization problem boils down to :

$$\max_{\tilde{\theta}_1} \Big\{ [\theta_1 - p_1^*] x_1(\tilde{\theta}_1) - \hat{R}(\tilde{\theta}_1) + \delta \int_{\hat{p}_2(\tilde{\theta}_1)}^{1} [1 - G_2(\theta_2|\theta_1)] d\theta_2 \Big\}.$$

As is well-known, second-order conditions are satisfied if an increase in $\tilde{\theta}_1$ does not decrease the derivative of the objective function with respect to θ_1, namely $x_1(\tilde{\theta}_1) - \delta \int_{\hat{p}_2(\tilde{\theta}_1)}^{1} (\frac{\partial G_2}{\partial \theta_1}) d\theta_2$. Sufficient conditions for this to be the case are :

a) $x_1(.)$ nondecreasing

b) $y(.)$ nondecreasing

c) $p_2(.)$ nonincreasing

d) $q_2(.)$ nonincreasing.

[Note that *b)*, *c)* and *d)* together imply that $\hat{p}_2(.)$ is nonincreasing.]

a) holds if $\theta_1 - \dfrac{\lambda}{1+\lambda} \dfrac{1 - F_1(\theta_1)}{f_1(\theta_1)}$ is nondecreasing

b) holds if the left-hand side of (16) is nondecreasing, which is the case if λ is not too large.

c) and *d)* hold if

$$\theta_2 - \frac{\lambda}{1+\lambda} \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \frac{\left(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)\right)}{g_2(\theta_2|\theta_1)}$$

is non decreasing in θ_1 for each θ_2.

An Example : $G_2(\theta_2|\theta_1) = 1 - e^{-\theta_2|\theta_1}$, $F_1(\theta_1) = \theta_1$

$$\begin{aligned} p_2(\theta_1) &= \frac{d_2\theta_1}{2\lambda\theta_1 + \theta_1 - \lambda} \\ q_2(\theta_1) &= 0 \text{ if } \qquad \theta_1 \geq \frac{\lambda}{1+2\lambda} \\ &= 1 \qquad\qquad \theta_1 < \frac{\lambda}{1+2\lambda} \end{aligned}$$

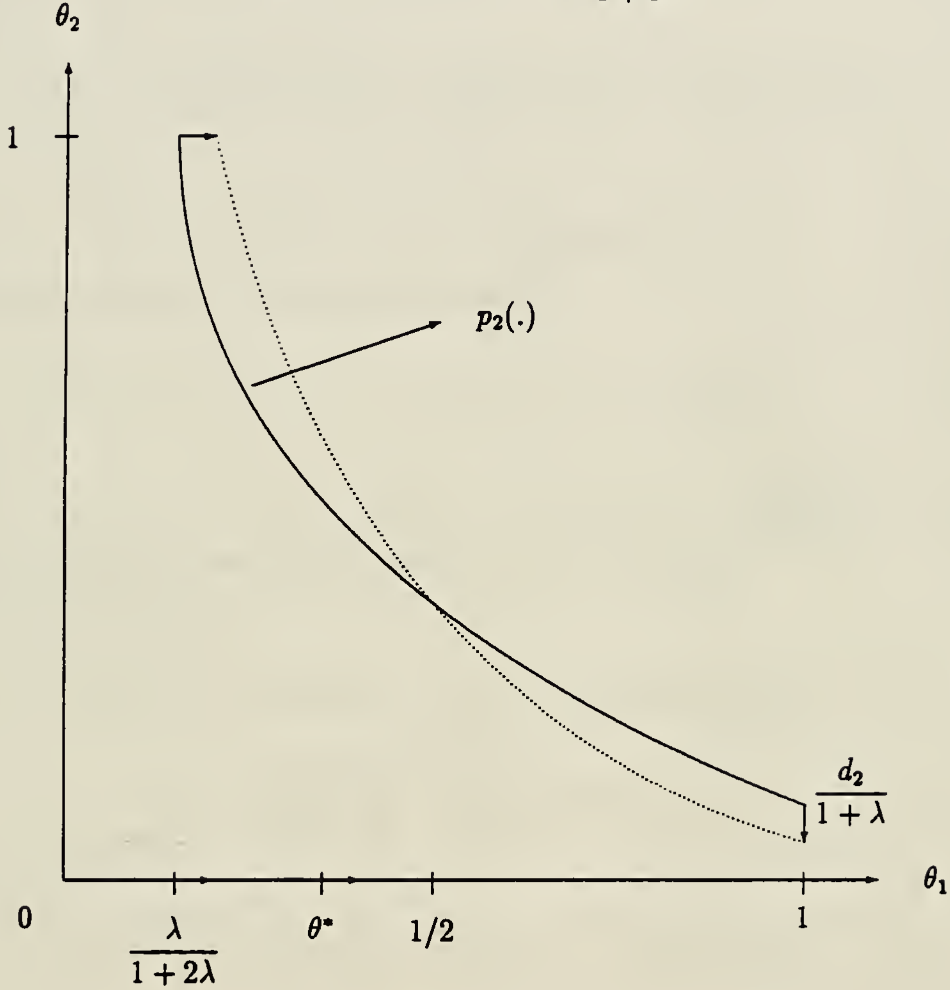


Figure 2

The dotted curve in figure 2 pictures the effect of an increase in the social cost of funds.

Turning to second-order conditions in period 2, the sufficient condition

$$\theta_2 - \frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\frac{-\frac{\partial G_2}{\partial \theta_1}}{g_2(\theta_2|\theta_1)}$$

nondecreasing in θ_2 takes the form

$$1 - \frac{\lambda}{1+\lambda}\frac{(1-\theta_1)}{\theta_1} > 0 \quad \text{or} \quad \theta_1 > \lambda.$$

For small θ_1 there is a problem illustrated by the bunching obtained in figure 2.

As for the second-order conditions in period 1, *c)* and *d)* always hold and *a)* and *b)* hold for λ small enough.

Q.E.D.

Appendix 3 : Proof that unbundled permits are more expansive than bundled ones under pure environmental regulation.

We look for a solution where $\forall\theta_1 \in [0, \theta^{\otimes}]$ the agent does not invest and asks for pollution rights in period 2, $\forall\theta_1 \in [\theta^{\otimes}, 1]$ the agent invests.

$\forall\theta_1 \in [0, \theta^{\otimes}]$

$$U_1(\theta_1) = \int_0^{\theta_1} \Big[x_1(\theta_1) + \delta \int_0^1 \Big[-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)x_2(\theta_1,\theta_2)\Big]d\theta_2\Big]d\theta_1$$

$\forall\theta_1 \in [\theta^{\otimes}, 1]$

$$U_1(\theta_1) = \int_0^{\theta_1} \Big[x_1(\theta_1) + \delta \int_0^1 \Big[-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)\Big]d\theta_1\Big]d\theta_1.$$

Optimizing social welfare as in section 5, we obtain

$$\forall\theta_1 \in [0,1] \quad x_1(\theta_1) = 1 \Leftrightarrow \theta_1 \geq \frac{d_1}{1+\lambda} + \frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)} \tag{A1}$$

$$\forall\theta_2 \in [0,\theta^{\otimes}] \quad x_2(\theta_1,\theta_2) = 1 \Leftrightarrow \theta_2 \geq \frac{d_2}{1+\lambda} + \frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\Big(\frac{-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta_1)}{g_2(\theta_2|\theta_1)}\Big). \tag{A2}$$

Let p_1^* and $p_2(\theta_1)$ the prices defined by equations (A1) and (A2).

Maximizing over $\theta^{\otimes}$ yields

$$-i + \delta\Big[\int_0^{p_2(\theta^{\otimes})} \theta_2 dG_2(\theta_2|\theta^{\otimes}) + \frac{d_2}{1+\lambda}(1 - G_2(p_2(\theta^{\otimes})|\theta^{\otimes}))\Big]$$

$$+\delta\frac{\lambda}{1+\lambda}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\int_0^{p_2(\theta^{\otimes})}\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta^{\otimes})d\theta_2 = 0.$$

Substituting from (A2)

$$-i + \delta\Big[\int_0^{p_2(\theta^{\otimes})} \theta_2 dG_2(\theta_2|\theta^{\otimes}) + \delta\big[p_2(\theta^{\otimes})(1 - G_2(p_2(\theta^{\otimes})|\theta^{\otimes}))\Big]$$

$$= \delta\frac{\lambda}{1+\lambda}\frac{1-F_1(\theta^{\otimes})}{f_1(\theta^{\otimes})}\Big\{\frac{-\frac{\partial G_2}{\partial \theta_1}(p_2(\theta^{\otimes})|\theta^{\otimes})}{g_2(p_2(\theta^{\otimes})|\theta^{\otimes})}.[1 - G_2(p_2(\theta^{\otimes})|\theta^{\otimes})]$$

$$+\int_0^{p_2(\theta^{\otimes})}\Big(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta^{\otimes})\Big)d\theta_2\Big\}$$

or

$$\Big\{\delta\int_0^1 \theta_2 dG_2(\theta_2|\theta^{\otimes}) - i\Big\} - \delta\int_{p_2(\theta^{\otimes})}^1 (\theta_2 - p_2(\theta^{\otimes}))dG_2(\theta_2|\theta^{\otimes})$$

$$= \delta\frac{\lambda}{1+\lambda}\frac{1-F_1(\theta^{\otimes})}{f_1(\theta^{\otimes})}\Big\{\frac{\big(-\frac{\partial G_2}{\partial \theta_1}(p_2(\theta^{\otimes})|\theta^{\otimes})\big)}{g_2(p_2(\theta^{\otimes})|\theta^{\otimes})}.(1 - G_2(p_2(\theta^{\otimes})|\theta^{\otimes}))$$

$$+\int_0^{p_2(\theta^{\otimes})}\Big(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta^{\otimes})\Big)d\theta_2 \geq 0. \tag{A3}$$

In terms of the implementation mechanism the indifference of agent $\theta^{\otimes}$ can be rewritten

$$\theta^{\otimes} - p_1^* - R(p_2(\theta^{\otimes})) + \delta \int_{p_2(\theta^{\otimes})}^{1} (\theta_2 - p_2(\theta^{\otimes}))dG_2(\theta_2|\theta^{\otimes})$$

$$= \theta^{\otimes} - p^{\otimes} + \delta \int_0^1 \theta_2 dG_2(\theta_2|\theta^{\otimes}) - i$$

or

$$p^{\otimes} - p_1^* = \left\{\delta \int_0^1 \theta_2 dG_2(\theta_2|\theta^{\otimes}) - i\right\} - \left\{\delta \int_{p_2(\theta^{\otimes})}^{1} (\theta_2 - p_2(\theta^{\otimes}))dG_2(\theta_2|\theta^{\otimes})\right\}$$
$$+R(p_2(\theta^{\otimes})) \geq 0.$$

from (A3) and $R(p_2(\theta^{\otimes})) \geq 0$.

Q.E.D.

Appendix 4 : Pure environmental pollution in the absence of first-period permits (case b of section 6).

Let A the (second-period) rent which is given up to type 0. The marginal type $\theta^{\otimes}$ is defined by (see figure 3, which depicts the agent's utility when he invests — steeper curve — and when he does not — flatter curve) :

$$A+\int_0^{\theta^{\otimes}}\Big(\int_{p_2(\theta_1)}^1\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)(\theta_2|\theta_1)d\theta_2\Big)d\theta_1=\int_0^{\theta^{\otimes}}\int_0^1\Big(-\frac{\partial G_2}{\partial\theta_1}(\theta_2|\theta_1)\Big)d\theta_2 d\theta_1$$

or

$$A=\int_0^{\theta^{\otimes}}\int_0^{p_2(\theta_1)}\Big(-\frac{\partial G_2}{\partial\theta_1}(\theta_2|\theta_1)\Big)d\theta_2 d\theta_1.$$

The cost of rents is now

$$\begin{aligned}
&- \lambda\int_0^{\theta^{\otimes}}\Big(A+\int_0^{\theta_1}\Big(\int_{p_2(\tilde\theta_1)}^1\Big(-\frac{\partial G_2}{\partial\theta_1}(\theta_2|\tilde\theta_1)\Big)d\theta_2 d\tilde\theta_1\Big)dF_1(\theta_1)\\
&- \lambda\int_{\theta^{\otimes}}^1\Big(\int_0^{\theta_1}\Big(\int_0^1\Big(-\frac{\partial G_2}{\partial\theta_1}(\theta_2|\tilde\theta_1)\Big)d\theta_2\Big)d\tilde\theta_1\Big)dF_1(\theta_1)=\\
&- \lambda F_1(\theta^{\otimes})\int_0^{\theta^{\otimes}}\int_0^{p_2(\theta_1)}\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1-\lambda F_1(\theta^{\otimes})\int_0^{\theta^{\otimes}}\int_{p_2(\theta_1)}^1\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1\\
&+ \lambda\int_0^{\theta^{\otimes}}F_1(\theta_1)\int_{p_2(\theta_1)}^1\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1-\lambda\int_{\theta}^1\int_0^1\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1\\
&+ \lambda F_1(\theta^{\otimes})\int_0^{\theta^{\otimes}}\int_0^1\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1+\lambda\int_{\theta^{\otimes}}^1F_1(\theta_1)\int_0^1\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1\\
&= -\lambda\int_0^1(1-F_1(\theta_1))\int_0^1\Big(-\frac{\partial G_2}{\partial\theta_1}(\theta_2|\theta_1)\Big)d\theta_2 d\theta_1\\
&- \lambda\int_0^{\theta^{\otimes}}F_1(\theta_1)\int_0^{p_2(\theta_1)}\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)(\theta_2|\theta_1)d\theta_2 d\theta_1.
\end{aligned}$$

The optimization problem is :

$$\begin{aligned}
\max\quad &- \delta D_2\Big(\int_0^{\theta^{\otimes}}\int_{p_2(\theta_1)}^1 dG_2(\theta_2|\theta_1)d\theta_2 dF_1(\theta_1)\Big)\\
&+ (1+\lambda)\delta\int_0^{\theta^{\otimes}}\int_{p_2(\theta_1)}^1\theta_2 dG_2(\theta_2|\theta_1)d\theta_2 dF_1(\theta_1)\\
&+ (1+\lambda)\delta\int_{\theta^{\otimes}}^1\Big[\int_0^1\theta_2 dG_2(\theta_2|\theta_1)-\frac{i}{\delta}\Big]dF_1(\theta_1)\\
&- \lambda\delta\int_0^1(1-F_1(\theta_1))\int_0^1\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1\\
&- \lambda\delta\int_0^{\theta^{\otimes}}F_1(\theta_1)\int_0^{p_2(\theta_1)}\Big(-\frac{\partial G_2}{\partial\theta_1}\Big)d\theta_2 d\theta_1.
\end{aligned}$$

We get :

$$\delta d_2 g_2(p_2(\theta_1)|\theta_1)f_1(\theta_1)-(1+\lambda)\delta p_2(\theta_1)g_2(p_2(\theta_1)|\theta_1)f_1(\theta_1)$$

$$-\lambda\delta F_1(\theta_1)\Big(-\frac{\partial G_2}{\partial \theta_1}\Big)(\theta_2|\theta_1) = 0$$

or

$$p_2(\theta_1) = \frac{d_2}{1+\lambda} - \frac{\lambda}{1+\lambda}\frac{F_1(\theta_1)}{f_1(\theta_1)}\frac{\big(-\frac{\partial G_2}{\partial \theta_1}\big)(p_2(\theta_1)|\theta_1)}{g_2(p_2(\theta_1)|\theta_1)} < \frac{d_2}{1+\lambda}.$$

Furthermore :

$$\begin{aligned}
&- \quad \delta d_2\Big(1 - G_2(p_2(\theta^{\otimes})|\theta^{\otimes})\Big)f_1(\theta^{\otimes}) + (1+\lambda)\delta\int_{p_2(\theta^{\otimes})}^{1}\theta_2 dG_2(p_2(\theta^{\otimes})|\theta^{\otimes})f_1(\theta^{\otimes}) \\
&- \quad (1+\lambda)\delta\int_0^1 \theta_2 dG_2(\theta_2|\theta^{\otimes})f_1(\theta^{\otimes}) + (1+\lambda)i f_1(\theta^{\otimes}) \\
&- \quad \lambda\delta\int_0^{p_2(\theta^{\otimes})}\Big(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta^{\otimes})\Big)d\theta_2 F_1(\theta^{\otimes}) = 0
\end{aligned}$$

or

$$\begin{aligned}
&-i + \delta\int_0^{p_2(\theta^{\otimes})}\theta_2 dG_2(\theta_2|\theta^{\otimes}) + \delta\frac{d_2}{1+\lambda}(1 - G_2(p_2(\theta^{\otimes})|\theta^{\otimes})) \\
&+\delta\frac{\lambda}{1+\lambda}\frac{F_1(\theta^{\otimes})}{f_1(\theta^{\otimes})}\int_0^{p_2(\theta^{\otimes})}\Big(-\frac{\partial G_2}{\partial \theta_1}(\theta_2|\theta^{\otimes})\Big)d\theta_2 = 0.
\end{aligned}$$

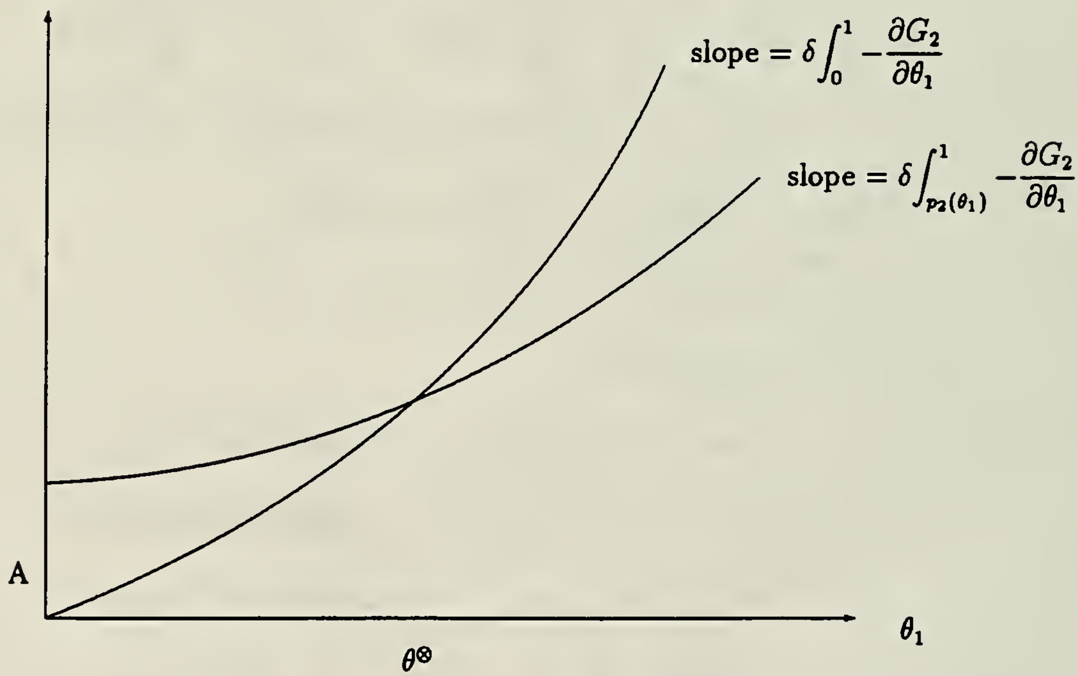


Figure 3

barcode p.34

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DEC. 05 1995		
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